THE EFFECT OF PRESSURE ON THE YOUNG'S MODULUS OF CERTAIN METALS*

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Investigation of the effect of high pressure on the variation in Young's modulus of various materials is not only of interest theoretically in defining more closely our theories concerning bonding forces in a material; it is also of great practical value, especially for certain fields of contemporary geophysics.

Hitherto no direct measurements have been made under pressure of Young's modulus as this presents technical difficulties [1]. The attention of investigators has thus been in the main directed towards studying the shear modulus. The variation with pressure in the shear modulus of metals was first studied using the static method by Bridgman [2], and a little later using the dynamic method [3 - 5].

In order to measure the change with pressure in Young's modulus E we designed and built special high-pressure equipment (up to 5,000 kg/cm^2 hydrostatic pressure, by means of which the pressure coefficient of Young's modulus could be measured directly using the static method. The principle of operation of the equipment is the variation in the rigidity of cantilevered test beam on the application of hydrostatic pressure. The rigidity was measured from the change in the angle of inclination of the test beam from the angle through which the high-pressure chamber must be turned in order to restore the initial deflexion of the test beam. Then for a constant deflexion of the beam the bending moment is directly proportional to the angle of rotation of the chamber. A detailed description of the equipment will be given in a later article.

The variation in Young's modulus with pressure was studied for metals (electrotechnical aluminium and copper with 0.05 – 0.15 per cent impurity contents, and medium-carbon 50 steel) of which the variation with pressure in the shear modulus G and the bulk modulus K were known from experiments. The test specimens were made in small batches of 5 – 6 units and were annealed at appropriate temperatures with subsequent slow cooling with the furnace.. The medium selected to convey high pressure to the specimen was a mixture of kerosine and transformer oil, the compressibility of which was also determined using the equipment.

The results of the experiments into the direct measurement of the coefficient of Young's modulus up to 4000 kg/cm^2 at room temperature are given in Table 1. The experimental

$$\frac{1}{E_0}\frac{dE}{dp}$$

values given are mean values of separate measurements made on three specimens. The measurements were made at 100 kg/cm^2 intervals both on increase and decrease in pressure. The maximum deviation of the measurements of the pressure coefficient of Young's modulus according to scatter of the experimental points was 10 per cent at a pressure of 4000 kg/cm^2, that is no more than 0.5 per cent of the Young's modulus itself.

The experimental data concerning the variation in the coefficient of Young's modulus with pressure obtained by means of direct measurement may be compared with calculated values of

$$\frac{1}{E_0}\frac{dE}{dp},$$

employing for this purpose experimental data available in the literature concerning the change with pressure of the bulk modulus K and the shear modulus G, and the well-known relationship between the elastic constants of an isotropic solid:

$$E = \frac{9KG}{3K + G}, \qquad (1)$$

that is valid in the case of small deformations (i.e. for not very large pressure intervals or for materials of low compressibility).

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TABLE 1.

Material	$\frac{1}{E_0}\frac{dE}{dp}$ ($10^6 cm^2/kg$)	
	According to equation (2)	Experimental data
Aluminium	7.15	7.2
Copper	2.69	4.3
Steel	2.24	2.3

Differentiating equation (1) and bearing in mind that in the range of pressures up to 5,000 kg/cm^2 it is possible with sufficient accuracy for practical purposes to substitute for the real values

$$\frac{1}{E}\frac{dE}{dp}, \quad \frac{1}{G}\frac{dG}{dp},$$

$$\frac{1}{K}\frac{dK}{dp} \quad \text{and} \quad \frac{G}{K}$$

their usual values

$$\frac{1}{E_0}\frac{dE}{dp}, \quad \frac{1}{G_0}\frac{dG}{dp}$$

$$\frac{1}{K_0}\frac{dK}{dp} \quad \text{and} \quad \frac{G_0}{K_0},$$

finally for the coefficient of Young's modulus we get:

$$\frac{1}{E_0}\frac{dE}{dp} \approx \frac{1}{3+\frac{G_0}{K_0}}\left(3\frac{1}{G_0}\frac{dG}{dp}+\frac{G_0}{K_0}\frac{1}{K_0}\frac{dK}{dp}\right), \quad (2)$$

where

$$\frac{1}{G_0}\frac{dG}{dp} \quad \text{and} \quad \frac{1}{K_0}\frac{dK}{dp}$$

are the coefficients of the shear and the bulk moduli respectively.

The relationship of the bulk modulus to the pressure can be calculated using Bridgman's semi-empirical formula that applies to many solids in the range of pressures up to 10,000 kg/cm^2

$$-\frac{\Delta V}{V} = ap - bp^2, \quad (3)$$

where a and b are material constants, of the orders of magnitude 10^{-7} cm^2/kg and 10^{-12} cm^4/kg^2 respectively; p is the pressure (in kg/cm^2).

From equation (3) in the zero approximation we get

$$\frac{1}{K_0}\frac{dK}{dp} \approx -a + \frac{2b}{a}. \quad (4)$$

The necessary experimental data concerning the magnitude:

$$E_0,\ K_0,\ G_0,\ \frac{1}{G_0}\frac{dG}{dp} \quad \text{and} \quad \frac{1}{K_0}\frac{dK}{dp}$$

for calculation according to equation (2) are given in Table 2. A comparison of the experimental values of the pressure coefficient obtained by us

$$\frac{1}{E_0}\frac{dE}{dp}$$

with the values calculated according to equation (2)

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TABLE 2.

Material	a, [1] $10^7 cm^2/kg$	b, [1] 10^{12} cm^4/kg^2	E_0, [3] 10^{-6} kg/cm^2	G_0 [3] 10^{-5} kg/cm^2	$K_0\ 10^{-6}$ kg/cm^2	$\frac{1}{G_0}\frac{dG}{dp}$[6] $10^6 cm^2/kg$	$\frac{1}{K_0}\frac{dK}{dp}$ $10^6 cm^2/kg$
Aluminium	13.40	3.44	0.72	2.67	0.74	7.61	3.74
Copper	7.16	1.04	1.30	4.77	1.39	2.71	2.18
Steel	5.83	0.80	2.13	8.26	1.70	2.36	2.11

is shown in Table 1. As will be seen from this table, in the range of pressures up to 4,000 kg/cm^2 the experimental data agree satisfactorily with the results obtained according to equation (2). The lack of agreement of the data for copper is evidently due to the fact that the modulus of normal elasticity of copper depends very extensively on the conditions of heat treatment of the specimens, that we were not able to reproduce accurately in our experiment owing to the absence of the necessary information in the literature [3-6].

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